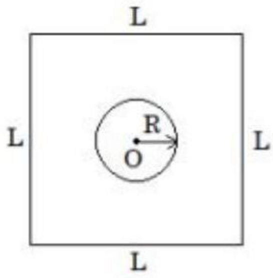
JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 29th January, 2023)	TIME : 9 : 00 AM to 12 : 00 NOON

Physics

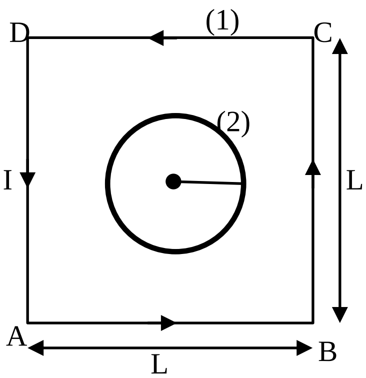
SECTION - A

1. Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side $(L \gg R)$. The loops are coplanar and their centers coincide :



(1) $M = \frac{\sqrt{2}\mu_0 R^2}{L}$ (2) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$ (3) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$ (4) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$

Sol. **(4)**



$\phi = MI$

$\phi_2 = MI_1$

$B_1A_2 = MI_1$

$$M = \frac{B_1A_2}{I_1} \quad \ldots.(1)$$

$B_1 \rightarrow$ magnetic field due to square frame

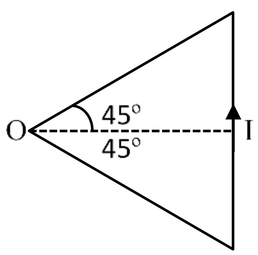
$A_2 \rightarrow$ Area of circle

$I_1 \rightarrow$ current in square frame.

$B_1 \rightarrow$

$B_1 = 4.\ B_{AB}$

$$= 4\left[\frac{\mu_0 I_1}{24\pi \frac{L}{2}}[\sin 45^\circ + \sin 45]\right]$$



$$B_1 = 2\frac{\mu_0 I_1}{\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 2\sqrt{2}\frac{\mu_0 I_1}{\pi L}$$

$$M = \frac{B_1 \cdot A_2}{I_1}$$

$$M = \left(\frac{2\sqrt{2}\mu_0 I_1}{\pi L}\right) \times \frac{\pi R^2}{I_1} = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

2. The threshold wavelength for photoelectric emission from a material is 5500A. Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a

(A) 75 W infra –red lamp
(B) 10 W infra-red lamp
(C) 75 W ultra – violet lamp
(D) 10 W ultra-violet lamp

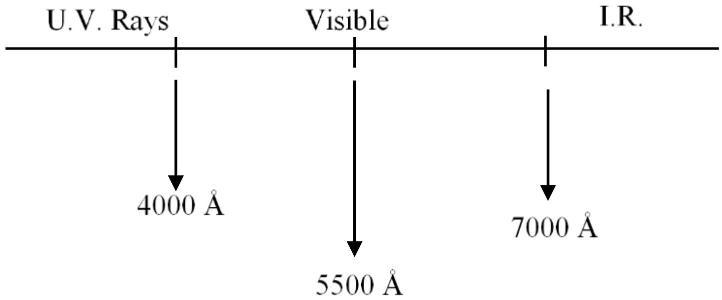
Choose the correct answer from the options given below:

(1) B and C only
(2) A and D only
(3) C only
(4) C and D Only

Sol. **(4)**

$$\lambda_0 = 5500\text{Å} \rightarrow \phi_0 = \frac{12400}{5500} = 2.25\,\text{eV}$$

$$\phi = 3.6 \times 10^{-19}\,\text{J}$$



- P.E.E will occur if wavelength of incidence wave is less then threshold wavelength. So u. v. rays will be useful for emission.
  So both U.V. rays lamps can be used.

3. Match List I with List II:

List I (Physical Quantity)	List II (Dimensional Formula)
A. Pressure gradient	I. $[M^0\ L^2\ T^{-2}]$
B. Energy density	II. $[M^1\ L^{-1}\ T^{-2}]$
C. Electric Field	III. $[M^1\ L^{-2}\ T^{-2}]$
D. Latent heat	IV. $[M^1\ L^1\ T^{-3}\ A^{-1}]$

Choose the correct answer from the options given below:

(1) A-II, B – III, C-I, D-IV
(2) A-II, B – III, C-IV, D-I
(3) A-III, B – II, C-IV, D-I
(4) A-III, B – II, C-I, D-IV

Sol. **(3)**

(A) Pressure gradient = $\frac{\text{Pressure}}{\text{Length}} = \frac{\text{Force}}{\text{Area} \times \text{length}}$

$= \frac{MLT^{-2}}{L^2 \cdot L} = [ML^{-2}T^{-2}]$

(B) Energy density = $\frac{\text{Energy}}{\text{Volume}} = \frac{ML^2T^{-2}}{L^3} = [M\,L^{-1}T^{-2}]$

(C) Electric field = $\frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{AT} = [M\,LT^{-3}\,A^{-1}]$

(D) Latent heat = $\frac{\text{Heat}}{\text{Mass}} = \frac{ML^2T^{-2}}{M} = [L^2\,T^{-2}]$

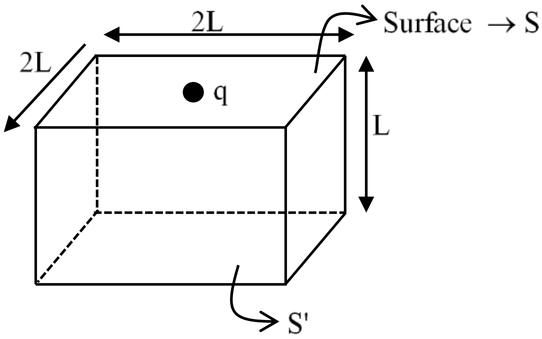
Ans : A-III, B-II, C-IV, D-I

Ans. : (3)

4. In a cuboid of dimension $2L \times 2L \times L$, a charge q is placed at the center of the surface ' S ' having area of $4L^2$. The flux through the opposite surface to ' S ' is given by

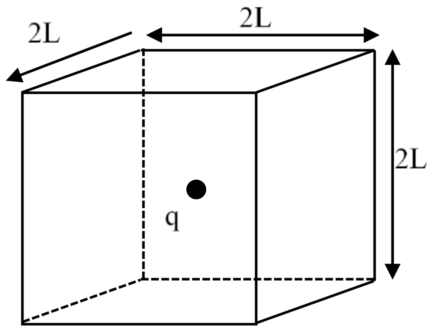
(1) $\frac{q}{12\varepsilon_0}$ (2) $\frac{q}{6\varepsilon_0}$ (3) $\frac{q}{3\varepsilon_0}$ (4) $\frac{q}{2\varepsilon_0}$

Sol. **(2)**



When smaller box is considered on the given box then charge 'q' will be at center.

So flux from surface S' = $\left(\frac{q}{\varepsilon_0}\right)\cdot\frac{1}{6} = \frac{q}{6\varepsilon_0}$



Ans : (2)

5. A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of 30 m/s. If both trains emit sounds with frequency 300 Hz, (Speed of sound: $\frac{330\text{ m}}{\text{s}}$) approximate difference of frequencies heard by the person will be:

(1) 55 Hz (2) 80 Hz (3) 33 Hz (4) 10 Hz

Sol. **(1)**

$\boxed{A} \rightarrow 30\,m/s$, $\boxed{\text{Observer}}$ $\boxed{B} \rightarrow 30\,m/s$

$f_0 = 300$ Hz

V = 330 m/sec.

$$f_A = f_0\left[\frac{V}{V - V_A}\right] = 300\left[\frac{330}{330-30}\right] = 330\text{Hz}$$

$$f_B = f_0\left[\frac{V}{V + V_A}\right] = 300\left[\frac{330}{360}\right] = 275\text{Hz}$$

$$\Delta f = f_A - f_B = 330 - 275 = 55\text{Hz}$$

Ans. : (1)

6. A block of mass m slides down the plane inclined at angle $30°$ with an acceleration $\frac{g}{4}$. The value of coefficient of kinetic friction will be:

(1) $\frac{1}{2\sqrt{3}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{2\sqrt{3}+1}{2}$ (4) $\frac{2\sqrt{3}-1}{2}$

Sol. **(1)**

$f_k = \mu N$

$N = mg\cos\theta$

$f_k = \mu\, mg\cos\theta$

$$a = \frac{mg\sin\theta - \mu mg\cos\theta}{m}$$

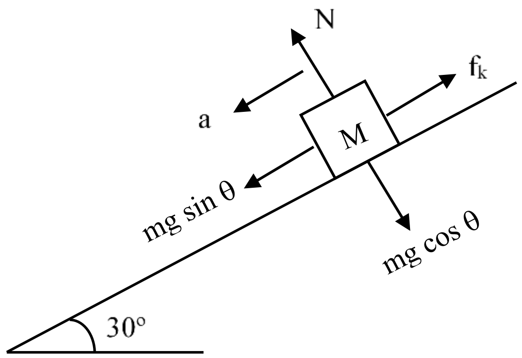
$a = g\sin 30^\circ - \mu g\cos 30^\circ$

$$\frac{g}{4} = g\left[\frac{1}{2} - \frac{\sqrt{3}\mu}{2}\right]$$

$$\frac{1}{2} = 1 - \sqrt{3}\mu$$

$$\sqrt{3}\mu = \frac{1}{2}$$

$$\boxed{\mu = \frac{1}{2\sqrt{3}}}$$



Ans. : 1

7. A bicycle tyre is filled with air having pressure of 270 kPa at 27°C. The approximate pressure of the air in the tyre when the temperature increases to 36° C is

(1) 270 kPa (2) 262 kPa (3) 360 kPa (4) 278 kPa

Sol. **(4)**

PV = nRT

n $\rightarrow$ const. V = const.

P α T,

P_1 = 270 kpa,

T_1 = 27ºC = 300 K

P_2 = ?,

T_2 = 36º = 36 + 273 = 309 K

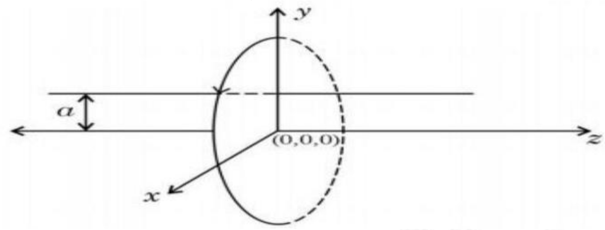
$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \quad \ldots(1)$$

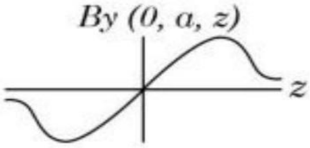
$$\frac{P_2}{270\text{ KPa}} = \frac{309}{300}$$

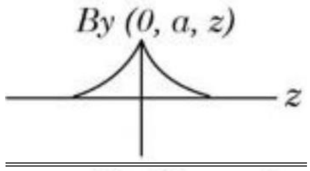
$$P_2 = \frac{103}{100} \times 270\text{KPa} \simeq 278\text{KPa}$$

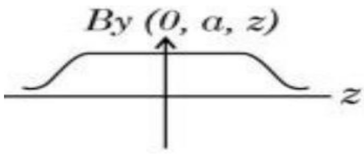
Option : (4)

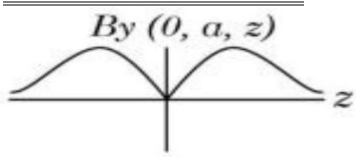
8. A single current carrying loop of wire carrying current I flowing in anticlockwise direction seen from +ve z direction and lying in xy plane is shown in figure. The plot of $\hat{j}$ component of magnetic field (By) at a distance $'a'$ (less than radius of the coil) and on yz plane vs z coordinate looks like



(1)


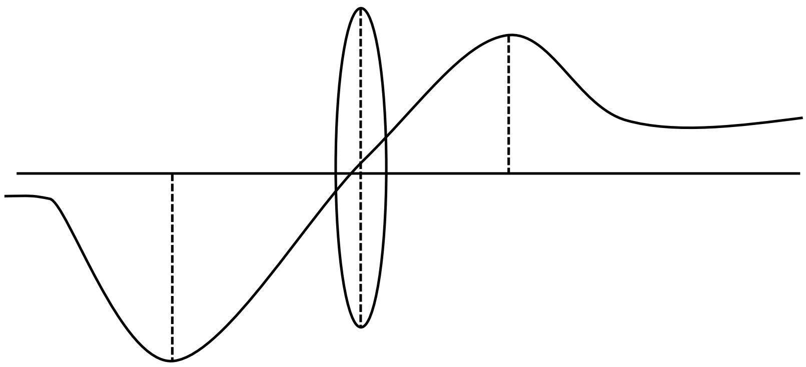
(2)


(3)


(4)


Sol. **(1)**

Theory based concept



9. Surface tension of a soap bubble is 2.0×10^{-2} Nm^{-1}. Work done to increase the radius of soap bubble from 3.5 cm to 7 cm will be:

Take $\left[\pi = \frac{22}{7}\right]$

(1) 9.24×10^{-4} J (2) 5.76×10^{-4} J (3) 0.72×10^{-4} J (4) 18.48×10^{-4} J

Sol. **(4)**

$T = 2.0 \times 10^{-2}$ Nm^{-1}

$r_1 = 3.5$ cm, $r_2 = 7$ cm

W = TΔA × No. of air – liquid surface

$$W = 2T.4\pi\left(r_2^2 - r_1^2\right)$$

$$W = 2 \times 2 \times 10^{-2} \times 4\pi\left[49 - \frac{49}{4}\right] \times 10^{-4}$$

$$W = 16\pi \times 10^{-6} \times 49 \times \frac{3}{4}$$

$W = 1847.26 \times 10^{-6}$

$W = 18.47 \times 10^{-4}$ J

10. Given below are two statements: One is labelled as Assertion **A** and the other is labelled as Reason **R**.

Assertion A: If

dQ and dW represent the heat supplied to the system and the work done on the system respectively. Then according to the first law of thermodynamics $dQ = dU - dW$.

Reason R: First law of thermodynamics is based on law of conservation of energy.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) A is not correct but R is correct

(3) A is correct but R is not correct

(4) Both A and R are correct but R is not the correct explanation of A

Sol. **(1)**

First law of thermodynamics is based on energy conservation

$dQ = dU + dW$

Here dW $\rightarrow$ work done on the system so volume decreases.

So dW $\rightarrow$ –ve

dQ = dU – dW

11. If a radioactive element having half-life of 30 min is undergoing beta decay, the fraction of radioactive element remains undecayed after 90 min. will be

(1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{16}$

Sol. **(1)**

T = 30 min.

t = 90 min

$$n = \frac{t}{T} = \frac{90\,\text{min}}{30\,\text{min}} = 3$$

$$N\ (\text{active}) = \frac{N_0}{2^n} = \frac{N_0}{2^3} = \frac{N_0}{8}$$

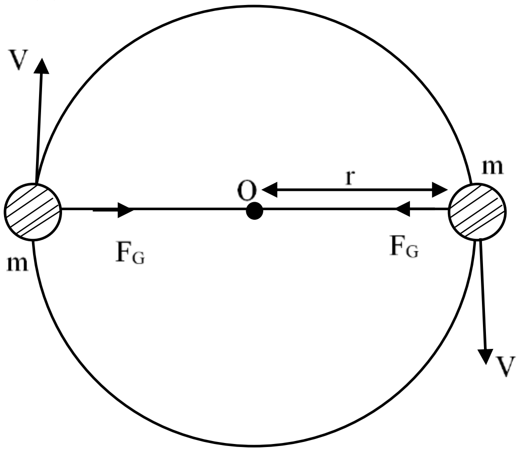
$$\boxed{\frac{N}{N_0} = \frac{1}{8}}$$

12. Two particles of equal mass 'm' move in a circle of radius 'r' under the action of their mutual gravitational attraction. The speed of each particle will be :

(1) $\sqrt{\frac{4Gm}{r}}$ (2) $\sqrt{\frac{Gm}{4r}}$

(3) $\sqrt{\frac{Gm}{r}}$ (4) $\sqrt{\frac{Gm}{2r}}$

Sol. **(2)**



$$\frac{mv^2}{r} = \frac{Gm.m}{(2r)^2}$$

$$\frac{v^2}{r} = \frac{Gm}{4r^2}$$

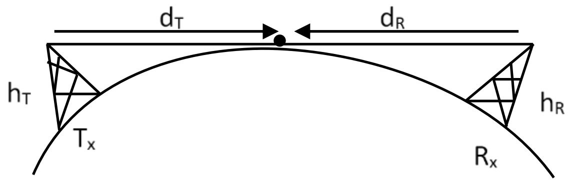
$$\boxed{V = \sqrt{\frac{Gm}{4r}}}$$

13. If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be: Given: Earth's radius = 6.4×10^6 m

(1) 28 km (2) 36 km (3) 32 km (4) 64 km

Sol. **(4)**

$h_T = h_R = h = 80$ m



$d_T = \sqrt{2R.h}$ and $d_R = \sqrt{2R.h}$

Maximum line of sight = $d_T + d_R$

$= \sqrt{2Rh} + \sqrt{2Rh}$

$= 2\sqrt{2Rh} = 2\sqrt{2 \times 6.4 \times 10^6 \times 80}$

$= 2\sqrt{64 \times 16 \times 10^6}$

$= 2 \times 8 \times 4 \times 10^3$

$= 64 \times 10^3 = 64$ km

14. A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [take $g = 10\ ms^{-2}$]

(1) 17 ms–1 (2) 13 ms^{-1} (3) 22.4 ms^{-1} (4) 3.4 ms^{-1}

Sol. **(2)**

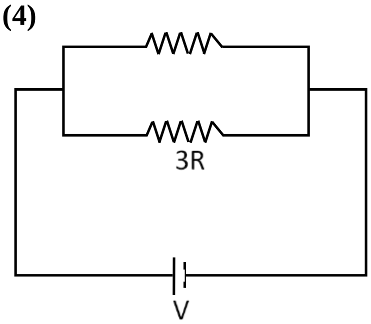
$\mu = 0.34$, R = 50 m

$V = \sqrt{\mu Rg} = \sqrt{0.34 \times 50 \times 10} = \sqrt{34 \times 5} = \sqrt{170} \simeq 13$

15. Ratio of thermal energy released in two resistors R and $3R$ connected in parallel in an electric circuit is :

(1) 1 : 27 (2) 1 : 1 (3) 1 : 3 (4) 3 : 1

Sol. **(4)**



$$H = I^2Rt = \frac{V^2}{R}.t$$

V = const.

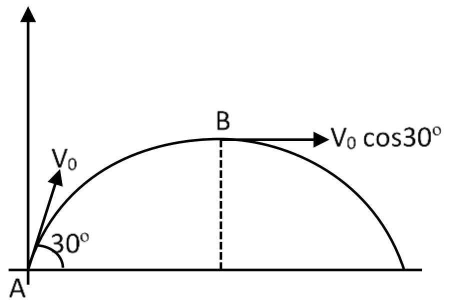
So, $H \alpha \frac{1}{R}$

$$\frac{H_1}{H_2} = \frac{3R}{R} = \frac{3}{1}$$

16. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be –

(1) 1 : 2 (2) 1 : 4 (3) 4 : 1 (4) 4 : 3

Sol. **(4)**



$$K_A = \frac{1}{2}mV_A^2$$

$$K_A = \frac{1}{2}mV_0^2 \qquad \ldots(1)$$

$$K_B = \frac{1}{2}m(V_0 \cos 30^o)^2$$

$$K_B = \frac{m}{2}.V_0^2 \frac{3}{4} = \frac{3}{8}mV_0^2 \qquad \ldots(2)$$

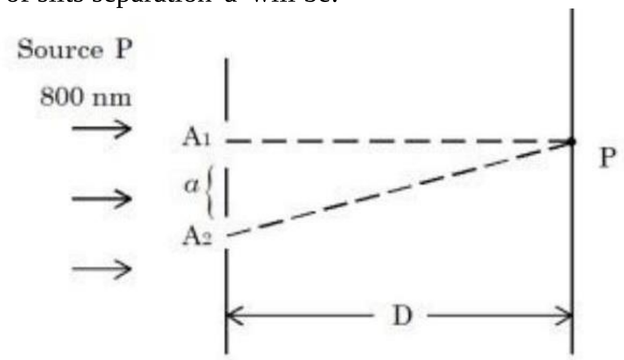
$$\frac{K_A}{K_B} = \frac{\left(\frac{mV_0^2}{2}\right)}{\left(\frac{3mV_0^2}{8}\right)}$$

$$\frac{K_A}{K_B} = \frac{4}{3}$$

17. Which of the following are true?
A. Speed of light in vacuum is dependent on the direction of propagation.
B. Speed of light in a medium is independent of the wavelength of light.
C. The speed of light is independent of the motion of the source.
D. The speed of light in a medium is independent of intensity.
Choose the correct answer from the options given below:
(1) C and D only (2) B and C only (3) A and C only (4) B and D only

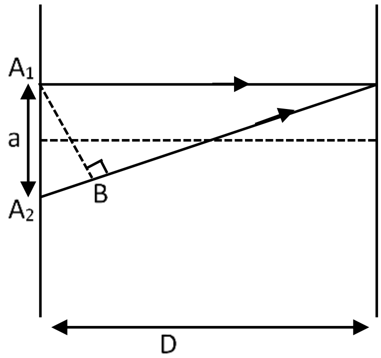
Sol. **(1)**
velocity of light depends on Refractive index of medium and independent of intensity and source.

18. In a Young's double slit experiment, two slits are illuminated with a light of wavelength 800 nm. The line joining A_1P is perpendicular to A_1A_2 as shown in the figure. If the first minimum is detected at P, the value of slits separation 'a' will be:



The distance of screen from slits D = 5 cm
(1) 0.5 mm (2) 0.1 mm (3) 0.4 mm (4) 0.2 mm

Sol. (4)



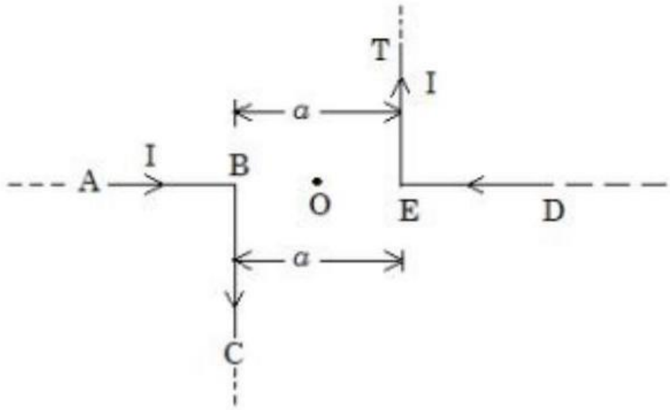
(1[st] minima) $\left(\frac{\beta}{2}\right)$

$$\frac{\beta}{2}=\frac{a}{2}$$
$$\boxed{\beta = a}$$
$$\frac{\lambda D}{a}=a$$
$$\lambda D = a^2$$
$$a^2 = 800\times10^{-9}\times5\times10^{-2}$$
$$a^2 = 4000\times10^{-11}$$
$$a = 2\times10^{-4}$$
$$\boxed{a = 0.2mm}$$

19. Which one of the following statement is not correct in the case of light emitting diodes?
A. It is a heavily doped p-n junction.
B. It emits light only when it is forward biased.
C. It emits light only when it is reverse biased.
D. The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.
Choose the correct answer from the options given below:
(1) A (2) C and D (3) C (4) B

Sol. **(3)**
Light emitting diode only used in forward bias
Option : 3

20. The magnitude of magnetic induction at mid point O due to current arrangement as shown in Fig will be



(1) $\frac{\mu_0 I}{\pi a}$ (2) $\frac{\mu_0 I}{2\pi a}$ (3) 0 (4) $\frac{\mu_0 I}{4\pi a}$

Sol. **(1)**
Magnetic field due to "AB" and "ED" will be zero
magnetic field due to "BC" and "ET" will be equal in amount and direction.

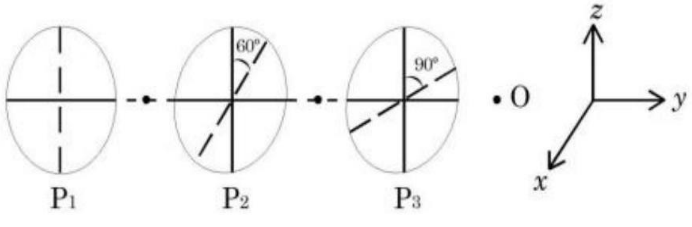
$$'B'\text{ due BC} = \frac{\mu_0 I}{4\pi r} = \frac{\mu_0 I}{4\pi \frac{a}{2}} = \frac{\mu_0 I}{2\pi a} \odot \qquad \ldots..(1)$$

$$'B'\text{ due to TE} = \frac{\mu_0 I}{2\pi a} \odot$$

$$B_{net}\text{ at point 'O'} = \left(\frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{2\pi a}\right) = \frac{\mu_0 I}{\pi a} \odot \text{ outward}$$

SECTION – B

21. As shown in the figure, three identical polaroids P_1, P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at angle of 60° and 90° with respect to axis of P_1.
The source S has an intensity of $256\frac{W}{m^2}$. The intensity of light at point O is – $\frac{W}{m^2}$.



Sol. **(24)**

Intensity of source $I_0 = 256\dfrac{W}{m^2}$

intensity after passing P_1 is $I_1 = \dfrac{I_0}{2} = 128\dfrac{W}{m^2}$

intensity after passing P_2 is $I_2 = I_1 \cos^2 \theta$

$= (128).\cos^2 60^o$

$128 \times \dfrac{1}{4} = 32\dfrac{W}{m^2}$

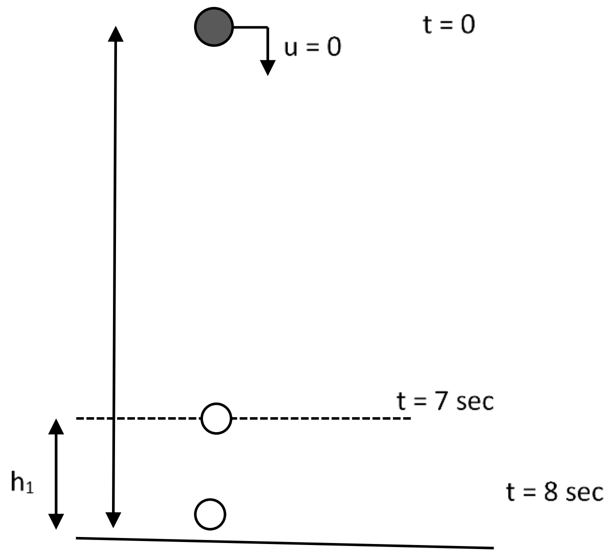
intensity after passing P_3 is $I_3 = I_2 \cos^2 \theta$

angle b/w p_2 and $p_3 = 30^o$

So, $I_3 = 32\cos^2 30^o = 32 \times \dfrac{3}{4} = 24\dfrac{W}{m^2}$

22. A 0.4 kg mass takes 8 s to reach ground when dropped from a certain height 'P' above surface of earth. The loss of potential energy in the last second of fall is J.
(Take $g = 10\ m/s^2$)

Sol. 300 J



$S = ut + \dfrac{1}{2}at^2$

$h = 0 + \dfrac{1}{2}.g(8)^2 = \dfrac{10}{2} \times 8 \times 8 = 320m$

Distance covered in last second

$h_1 = u + \dfrac{a}{2}(2n-1)$

$= 0 + \dfrac{10}{2}[2(8)-1]$

$h_i = 5[15] = 75m$

$\Delta U_{loss} = mg\Delta h$

$\Delta U_{loss} = 0.4 \times 10 \times 75 = 300J$

Ans $\rightarrow$ 300 J

23. Two simple harmonic waves having equal amplitudes of 8 cm and equal frequency of 10 Hz are moving along the same direction. The resultant amplitude is also 8 cm. The phase difference between the individual waves is ________degree.

Sol. **120**

$A_1 = A \quad A_2 = A \quad A_{eq} = A$

$A_1^2 + A_2^2 + 2A_1A_2\cos\phi = A_{eq}^2$

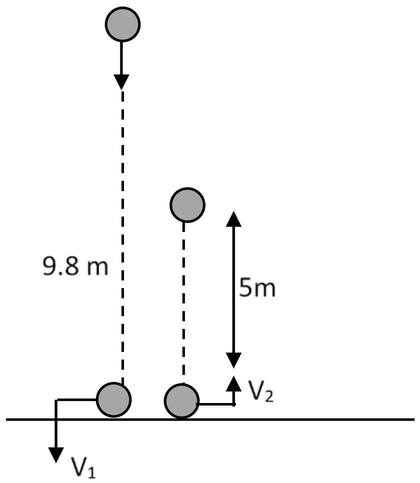
$A^2 + A^2 + 2A^2\cos\phi = A^2$

$1 + 2\cos\phi = 0 \Rightarrow \cos\phi = -\frac{1}{2}$

$\phi = 120$

24. A tennis ball is dropped on to the floor from a height of 9.8 m. It rebounds to a height 5.0 m. Ball comes in contact with the floor for 0.2 s. The average acceleration during contact is ms^{-2}
(Given $g = 10\ ms^{-2}$)

Sol. **(120m / sec^2)**



$v_1 = \sqrt{2gh} = \sqrt{2\times10\times9.8} = \sqrt{196}$

$v_1 = 14 m/sec$

$v_2 = \sqrt{2gh}$

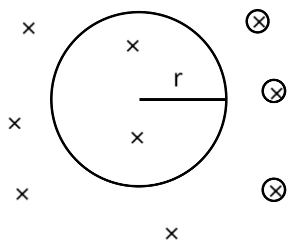
$v_2 = \sqrt{2\times10\times5} = 10 m/sec.$

$a_{aug} = \frac{\Delta v}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{10-(-14)}{0.2}$

$a_{ay} = \frac{24}{0.2} = 120 m/sec^2$

25. A certain elastic conducting material is stretched into a circular loop. It is placed with its plane perpendicular to a uniform magnetic field $B = 0.8$ T. When released the radius of the loop starts shrinking at a constant rate of $2 cms^{-1}$. The induced emf in the loop at an instant when the radius of the loop is 10 cm will be _____ mV.
(Given $g = 10\ ms^{-2}$)

Sol. **(10)**



$B = 0.8T$

$\frac{dr}{dt} = 2cms^{-1}$

$emf = \frac{d\phi}{dt} = \frac{d(BA)}{dt}$

$emf = B\frac{d}{dt}\pi r^2 = \pi B(2r)\frac{dr}{dt}$

$emf = 2\pi Br \cdot (0.02)$

$= 2\pi(0.8)(0.1) \times 0.02$

$= 32\pi \times 10^{-4}$

$= 100.48 \times 10^{-4}$

$= 10.048 \times 10^{-3}$

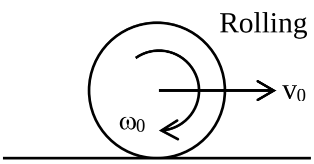
$= 10.04mV \approx 10\text{ mV}$

26. A solid sphere of mass 2 kg is making pure rolling on a horizontal surface with kinetic energy 2240 J. The velocity of centre of mass of the sphere will be _____ ms^{-1}

Sol. **(40)**

Mass = 2 kg

K.E = 2240 J



K.E = $\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2$

$= \frac{1}{2}mv_0^2 + \frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\frac{v_0^2}{R^2}$

$= \frac{1}{2}mv_0^2 + \frac{mv_0^2}{5}$

K.E = $\frac{7}{10}mv_0^2$

$2240 = \frac{7}{10} \times 2 \times v_0^2$

$v_0^2 = \frac{22400}{14} = 1600$

$\boxed{v_0 = 40\text{ m / sec}}$

27. A body cools from 60°C to 40°C in 6 minutes. If, temperature of surroundings is 10°C. Then, after the next 6 minutes, its temperature will be °C.

Sol. **(28)**

$60°C \xrightarrow[6\text{ min}]{} 40°C \xrightarrow[6\text{ min}]{} T \qquad T_0 = 10°C$

$\frac{\Delta T}{\Delta t} = k(T - T_0)$

$\frac{(60-40)}{6\min} = k[50-10] \qquad \ldots(1)$

And $\frac{(40-T)}{6\min} = K\left[\frac{40+T}{2} - 10\right] \qquad \ldots(2)$

(1) / (2)

$\frac{20}{40-T} = \frac{40}{\left(\frac{40+T-20}{2}\right)}$

$$\frac{20}{40-T}=\frac{40\times 2}{20+T}$$

(20 + T) = (40 – T)4

20 + T = 160 – 4T ⇒ ST = 140

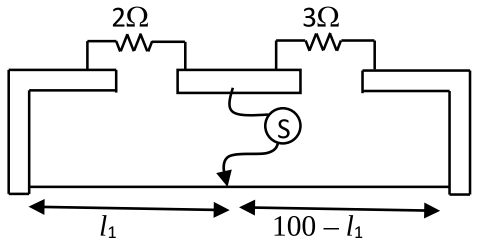
$$T = \frac{140}{5} = 28°C$$

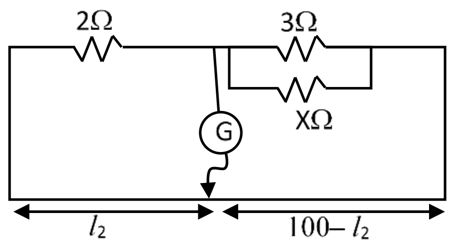
28. In a metre bridge experiment the balance point is obtained if the gaps are closed by 2Ω and 3Ω. A shunt of XΩ is added to 3Ω resistor to shift the balancing point by 22.5 cm. The value of X is -

Sol. **x = 2**

$$\frac{2}{\ell_1}=\frac{3}{100-\ell_1}$$

$200 - 2\ell_1 = 3\ell_1$





$200 = 5\ell_1$

$\boxed{\ell_1 = 40\,cm}$

Now $\ell_2 = \ell_1 + 22.5$

$\ell_2 = 40 + 22.5 = 62.5$ cm

$$\text{So, } \frac{2}{62.5}=\frac{\left(\frac{3\cdot x}{3+x}\right)}{37.5} \Rightarrow (37.5)\times 2=\frac{(62.5)(3x)}{3+x}$$

$$3+x=\frac{(62.5)}{25}x$$

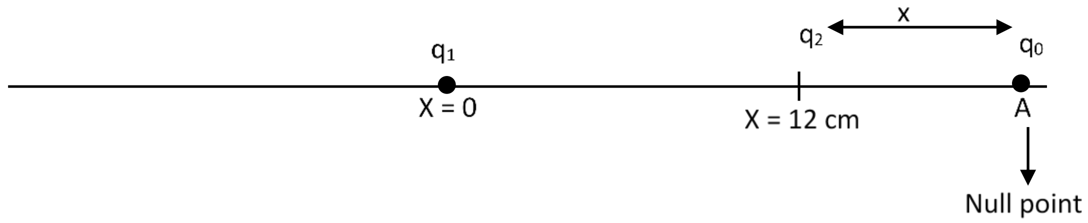
3 + x = 2.5 x

$3 = 1.5x \Rightarrow \boxed{x=2}$

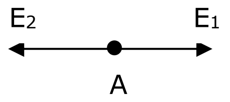
29. A point charge $q_1 = 4q_0$ is placed at origin. Another point charge $q_2 = -q_0$ is placed at = 12 cm. Charge of proton is q_0. The proton is placed on x axis so that the electrostatic force on the proton is zero. In this situation, the position of the proton from the origin is _________ cm.

Sol. **24**

$q_1 = 4q_0 \text{ and } q_2 = -q_0$



Electric field at point A will be zero.



$\left|\vec{E_1}\right| = \left|\vec{E_2}\right|$

$\dfrac{kq_1.q_0}{(12+x)^2} = \dfrac{kq_2.q_0}{x^2}$

$\dfrac{4q_0}{(12+x)^2} = \dfrac{q_0}{x^2}$

$4x^2 = (12 + x)^2$

$\pm 2x = (12 + x)$

$2x = 12 + x$	$-2x = 12 + x$
$x = 12$	$-3x = 12$
$x = 12$ cm	$x = x = -\dfrac{12}{3} = -4$

Position of proton from origin will be $\rightarrow 12 + 12$
$\rightarrow 24$cm

30. A radioactive element ${}^{242}_{92}X$ emits two α –articles, one electron and two positrons. The product nucleus is represented by ${}^{234}_{P}Y$. The value of P is

Sol. **(87)**

${}_{92}X^{242} \longrightarrow {}_{P}Y^{234} + 2_2\alpha^4 + {}_{-1}e^0 + 2_{+1}e^0$

Using charge conservation:

92 = P + 2(2) + (-1) + 2(1)

92 = P + 5

$\boxed{P = 87}$ Ans.

Chemistry

SECTION - A

31. "A" obtained by Ostwald's method involving air oxidation of NH_3, upon further air oxidation produces "B". "B" on hydration forms an oxoacid of Nitrogen along with evolution of "A". The oxoacid also produces "A" and gives positive brown ring test.
Identify A and B, respectively.
(1) N_2O_3, NO_2 (2) $NO_2,\ N_2O_4$ (3) $NO_2,\ N_2O_5$ (4) NO, NO_2

Sol. 4

$$4\,NH_3 + 5O_2 \xrightarrow{\Delta} \underset{(A)}{4NO} + 6H_2O$$

$$2NO + O_2 \rightarrow \underset{(B)}{2NO_2}$$

32. Correct statement about smog is:
(1) Classical smog also has high concentration of oxidizing agents
(2) Both NO_2 and SO_2 are present in classical smog
(3) NO_2 is present in classical smog
(4) Photochemical smog has high concentration of oxidizing agents

Sol. 4
Photochemical smog is oxidizing smog. Its high concentration of oxidizing agent like ozone and HNO_3

33. The standard electrode potential (M^{3+}/M^{2+}) for V, Cr, Mn& Co are -0.26 V, -0.41 V, $+1.57$ V and $+1.97$ V, respectively. The metal ions which can liberate H_2 from a dilute acid are
(1) Mn^{2+} and Co^{2+} (2) Cr^{2+} and Co^{2+} (3) V^{2+} and Cr^{2+} (4) V^{2+} and Mn^{2+}

Sol. 3
V^{+2} and Cr^{+2}
The metal ion for which have less value of reduction potential can release H_2 on reaction with dilute acid.

34. The shortest wavelength of hydrogen atom in Lyman series is λ. The longest wavelength in Balmer series of $He^{+}$is
(1) $\frac{36\lambda}{5}$ (2) $\frac{9\lambda}{5}$ (3) $\frac{5}{9\lambda}$ (4) $\frac{5\lambda}{9}$

Sol. 2

For lymen seriese $\rightarrow \frac{1}{\lambda_{min}} = R \times 1\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

For balmer seriese $\rightarrow \frac{1}{\lambda_{max}} = R \times 4\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$$\frac{\frac{1}{\lambda_{min}}}{\frac{1}{\lambda_{max}}} = \frac{\lambda_{max}}{\lambda_{min}} = \frac{\lambda_{max}}{\lambda} = \frac{9R}{5R}$$

$$\lambda_{max} = \frac{9\lambda}{5}$$

35. The bond dissociation energy is highest for
(1) F_2 (2) Br_2 (3) I_2 (4) Cl_2

Sol. 4

Order of B.D.E in halogen is

(E) Cl–Cl > Br-Br > F-F > I-I

36. The increasing order of pK_a for the following phenols is

(A) 2, 4-Dinitrophenol (B) 4-Nitrophenol

(C) 2, 4,5 – Trimethylphenol (D) Phenol

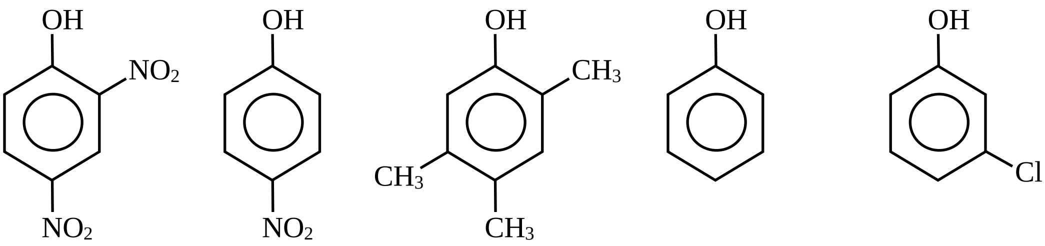
(E) 3-Chlorophenol

Choose the correct answer from the option given below:

(1) (A), (B), (E), (D), (C) (2) (C), (D), (E), (B), (A)

(3) (A), (E), (B), (D), (C) (4) (C), (E), (D), (B), (A)

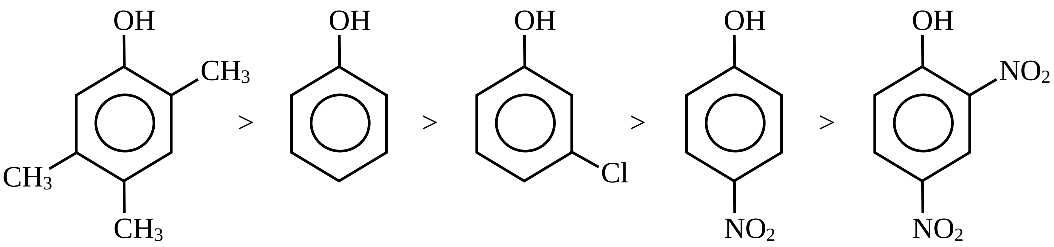
Sol. 1



(A) (B) (C) (D) (E)

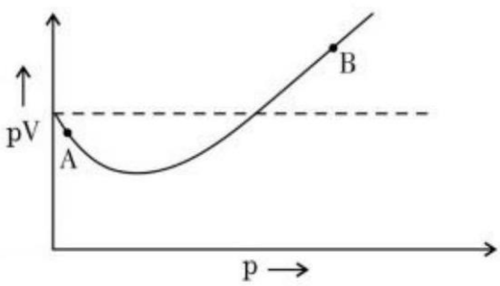
acedic strength $\propto K_a$

$\propto \frac{1}{PK_a}$



(C) (D) (E) (B) (A)

37. For 1 mol of gas, the plot of pV vs. p is shown below. p is the pressure and V is the volume of the gas



What is the value of compressibility factor at point ?

(1) $1 + \frac{a}{RTV}$ (2) $1 - \frac{a}{RTV}$ (3) $1 + \frac{b}{V}$ (4) $1 - \frac{b}{V}$

Sol. 2

At point A $\rightarrow$ low pressure, volume of gas very high

$\rightarrow$ V–b $\approx$ V

$$\left(p+\frac{a}{V^2}\right)\left(v-\underset{\text{neglect}}{b}\right)=RT$$

$$\left(p+\frac{a}{V^2}\right)v=RT$$

$$PV+\frac{a}{v}=RT$$

$$z+\frac{a}{RTV}=1$$

$$z=1-\frac{a}{RTV}$$

38. Match List I with List II.

List I	List II
Antimicrobials	Names
(A) Narrow Spectrum Antibiotic	(I) Furacin
(B) Antiseptic	(II) Sulphur dioxide
(C) Disinfectants	(III) Penicillin G
(D) Broad spectrum antibiotic	(IV) Chloramphenicol

Choose the correct answer from the options given below:

(1) (A) – II, (B) – I, (C) – IV, (D) – III
(2) (A) –I, (B) – II, (C) – IV, (D) – III
(3) (A) – II, (B) – I, (C) – IV, (D) – II
(4) (A) –III, (B) – I, (C) – II, (D) – IV

Sol. 4

Narrow Spectrum Antibiotic $\rightarrow$ Penicillin G (used in pathgens)

Antiseptic $\rightarrow$ Furacin

Disinfectants $\rightarrow$ Sulphur dioxide

Broad spectrum antibiotic $\rightarrow$ Chloramphenicol

39. During the borax bead test with $CuSO_4$, a blue green colour of the bead was observed in oxidising flame due to the formation of

(1) CuO (2) $Cu(BO_2)_2$ (3) $Cu_3\ B_2$ (4) Cu

Sol. 2

Blue green colour is due to formation of $Cu(BO_2)_2$

$CuSO_4 \xrightarrow{\Delta} CuO+SO_3$

$CuO+B2O_3 \rightarrow Cu(BO_2)_2$

40. Which of the following salt solution would coagulate the colloid solution formed when $FeCl3$ is added to NaOH solution, at the fastest rate?

(1) 10 mL of 0.1 mol dm^{-3} Na_2SO_4 (2) 10 mL of 0.2 mol dm^{-3} $AlCl_3$

(3) 10 mL of 0.1 mol dm^{-3} $Ca_3(PO_4)_2$ (4) 10 mL of 0.15 mol dm^{-3} $CaCl_2$

Sol. 2

$FeCl_3 + NaOH \rightarrow Fe(OH)_3/OH^-$

Negative colloidal particle

Positive ion required for coagulation of sol.

41. Number of cyclic tripeptides formed with 2 amino acids A and B is:

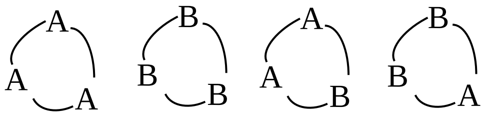
(1) 5 (2) 2 (3) 4 (4) 3

Sol. 3

To amine acid

$$\underset{\text{(A)}}{H_2N-\underset{\underset{R_1}{|}}{CH}-COOH} \qquad \underset{\text{(B)}}{H_2N-\underset{\underset{R_2}{|}}{CH}-COOH}$$

Tripeptide are formed $\rightarrow$



42. The correct order of hydration enthalpies is

(A) K^+ (B) Rb^+ (C) Mg^{2+} (D) Cs^+

(E) Ca^{2+}

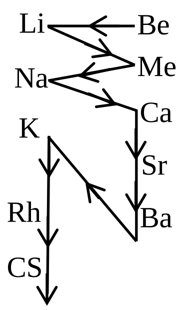
Choose the correct answer from the options given below:

(1) $E > C > A > B > D$ (2)$C > A > E > B > D$

(2) $C > E > A > D > B$ (4) $C > E > A > B > D$

Sol. 4

Order of hydration enthalpy is size order



$Mg^{2+} > Ca^{2+} > K^+ > Rb^+ > CS^+$

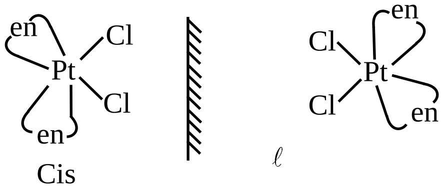
43. Chiral complex from the following is:

Here en = ethylene diamine

(1) cis $^-[PtCl_2(en)_2]^{2+}$ (2) trans $-[PtCl_2(en)_2]^{2+}$

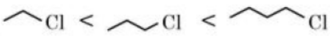
(3) cis $-[PtCl_2(NH_3)_2]$ (4) trans $-[Co(NH_3)_4Cl_2]^+$

Sol. 1

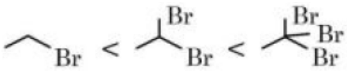


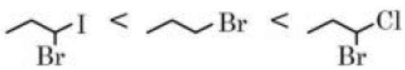
d Cis ℓ

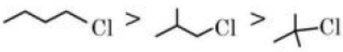
44. Identify the correct order for the given property for following compounds.

(A) Boiling Point: 

(B) Density: 

(C) Boiling Point: 

(D) Density: 

(E) Boiling Point: 

Choose the correct answer from the option given below:

(1) (B), (C) and (D) only
(2) (A), (C) and (D) only
(3) (A), (B) and (E) only
(4) (A), (C) and (E) only

Sol. 4

(i) B.P. $\propto$ Molecular mass

(ii) B.P. $\propto$ polarity$\uparrow$

(iii) B.P. $\propto \dfrac{1}{\text{No.of Branches}}$

45. The magnetic behavior of Li_2O, Na_2O_2 and KO_2, respectively, are

(1) Paramagnetic, paramagnetic and diamagnetic
(2) diamagnetic, paramagnetic and diamagnetic
(3) paramagnetic, diamagnetic and paramagnetic
(4) diamagnetic, diamagnetic and paramagnetic

Sol. 4

Li_2O	O^{--}	Diamagnetic
Na_2O_2	O_2^{--}	Diamagnetic
KO_2	O_2^{-}	paramagnetic

46. The reaction representing the Mond process for metal refining is____________

(1) $ZnO + C \xrightarrow{\Delta} Zn + CO$

(2) $Zr + 2I_2 \xrightarrow{\Delta} ZrI_4$

(3) $2\,K[Au(CN)_2] + Zn \xrightarrow{\Delta} K_2[Zn(CN)_4] + 2Au$

(4) $Ni + 4CO \xrightarrow{\Delta} Ni(CO)_4$

Sol. 4

$$\underset{\text{Impure}}{Ni+4CO} \xrightarrow{50^{o}C} \underset{\text{vap.}}{Ni(CO)_4}$$

$$Ni(CO)_4 \xrightarrow{250^{o}C} \underset{\text{pure}}{Ni + 4CO}$$

47. Which of the given compounds can enhance the efficiency of hydrogen storage tank?

(1) Di-isobutylaluminium hydride (2) $NaNi_5$
(3) Li/P_4 (4) SiH_4

Sol. 2

Ni can adsorb 800 times more hydrogen then its own volume

48. Match List I with List II.

List I		List II
Reaction		Reagents
(A)	Hoffmann Degradation	(I) Conc.KOH, Δ
(B)	Clemenson reduction	(II) $CHCl_3$, $NaOH/H_3O^{\oplus}$
(C)	Cannizaro reaction	(III) Br_2, NaOH
(D)	Reimer-Tiemann Reaction	(IV) Zn − Hg/HCl

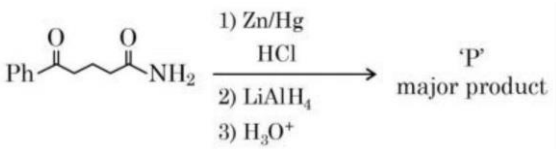
Choose the correct answer from the options given below:

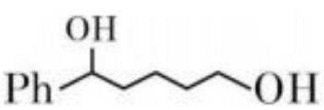
(1) (A) –III, (B) –IV, (C) – I, (D) – II (2) (A) - II, (B) –I, (C) - III, (D) – IV
(3) (A) –III, (B) –IV, (C) – II, (D) – I (4) (A) –II, (B) – IV, (C) – I, (D) – III

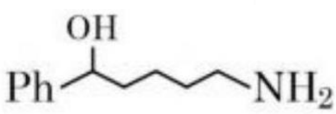
Sol. 1

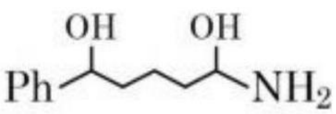
Hoffmann degradation → Br_2, NaOH
Clemenson reduction → Zn-Hg/HCl
Cannizaro reaction → Conc. KOH, Δ
Reimer-Tiemann reaction → $CuCl_3$, $NaOH/H_3O^{\oplus}$

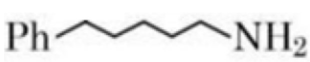
49. The major product 'P' for the following sequence of reactions is:



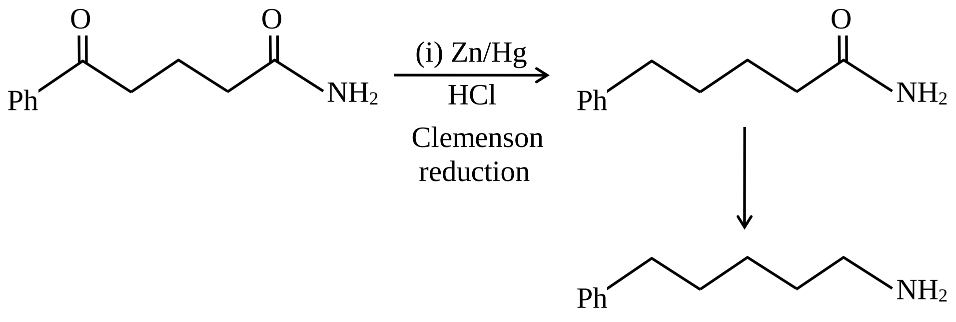
(1) 

(2) 

(3) 

(4) 

Sol. 4



50. Compound that will give positive Lassaigne's test for both nitrogen and halogen is:
(1) $NH_2OH.HCl$ (2) $CH_3NH_2.HCl$ (3) NH_4Cl (4) $N_2H_4.HCl$

Sol. 2
Lassaigne test for both N and X is given by the compound which have C, N as well X atom in compound.

51. Millimoles of calcium hydroxide required to produce 100 mL of the aqueous solution of pH 12 is $x \times 10^{-1}$. The value of x is________(Nearest integer).
Assume complete dissociation.

Sol. 5
pH=12, pOH=2 $[OH^-]=10^{-2}$ N

Molarity of $Ca(OH)_2 = \frac{N}{2} = \frac{10^{-2}}{2} = 0.005$ N

$0.005 = \frac{\text{mili moles}}{100}$

$= \frac{5}{1000} = \frac{\text{mili moles}}{100}$

$= 5\times10^{-1}$ milimoles

52. Water decomposes at 2300 K
$H_2O(g) \rightarrow H_2(g) + \frac{1}{2}O_2(g)$
The percent of water decomposing at 2300 K and 1 bar is________(Nearest integer).
Equilibrium constant for the reaction is 2×10^{-3} at 2300 K.

Sol. 2
$H_2O(g) \rightarrow H_2(g) + 1/2O_2$
$1-\propto \quad \propto \quad \propto/2$

$k_p = \frac{\propto(\propto/2)^{1/2}}{1-\propto} = 2\times10^{-3}$

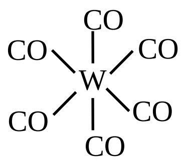
$2\times10^{-3} = \frac{\propto^{3/2}}{\sqrt{2}(1-\propto)}$ $\quad \propto \ll 1$

$2^{3/2}\times(10^{-2})^{3/2} = \propto^{3/2}$
$\propto = 2\times10^{-2}$

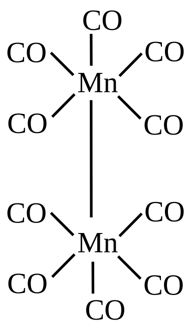
53. The sum of bridging carbonyls in $W(CO)_6$ and $Mn_2(CO)_{10}$ is________

Sol. 0

$W(CO)_6 \rightarrow$ 0 Bridge CO



$Mn_2(CO)_{10} \rightarrow 0$



54. Solid Lead nitrate is dissolved in 1 litre of water. The solution was found to boil at 100.15^0C. When 0.2 mol of NaCl is added to the resulting solution, it was observed that the solution froze at -0.8^0C. The solubility product of $PbCl_2$ formed is______$\times 10^{-6}$ at 298 K. (Nearest integer)
(Given : K_b=0.5 K kgmol^{-1} and $K_f = 1.8$ K kg mol^{-1}. Assume molality to be equal to molarity in all cases.)

Sol. 13

Let a mole Pb $(NO_3)_2$ be added

$$Pb(NO_3)_2 \rightarrow Pb^{2+} + 2NO_3^-$$

a a 2a

$$\Delta T_b = 0.15 = 0.5[3a] \Rightarrow a = 0.1$$

$$Pb^{2+}_{(aq)} + 2Cl^-_{(aq)} \rightarrow PbCl_2(s)$$

t = 0	0.1	0.2
t = ∞	(0.1 – x)	(0.2 – 2x)

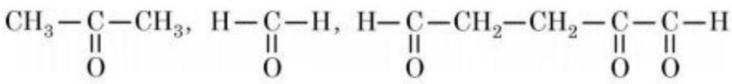
In final solution

$$\Delta T_f = 0.8 = 1.8\left[\frac{0.3+3x+0.2+0.2}{1}\right]$$

$$\Rightarrow x = \frac{2.3}{27}$$

$$\Rightarrow K_{sp} = \left(0.1-\frac{2.3}{27}\right)\left(0.2-\frac{4.6}{27}\right)^2 = 13\times10^{-6}$$

55. 17mg of a hydrocarbon (M.F. $C_{10}H_{16}$) takes up 8.40 mL of the H_2 gas measured at 0°C and 760 mm of Hg. Ozonolysis of the same hydrocarbon yields



The number of double bond/s present in the hydrocarbon is________

Sol. 3

Moles of hydrocarbon = $\frac{17\times10^{-3}}{136} = 1.25\times10^{-4}$

$nH_2 = 1\times\frac{8.4}{1000} = n\times0.0821\times273$

$\Rightarrow\ n = 3.75\times10^{-4}$

Hydrogen molecule used for 1 molecule of hydrogen is 3

$= \frac{3.75\times10^{-4}}{1.25\times10^{-4}} = 3$

56. Consider the following reaction approaching equilibrium at 27°C and 1 atm pressure

$A + B \underset{k_r=10^2}{\overset{k_f=10^{\ 3}}{\rightleftharpoons}} C + D$

The standard Gibb's energy change $\left(\Delta_r G^\theta\right)$ at 27°C is (–)_______KJ mol^{-1}

(Nearest integer).

(Given: $R = 8.3\ J\ K^{-1}\ mol^{-1}$ and $\ln 10 = 2.3$)

Sol. 6

$K_{eq} = \frac{K_f}{K_b} = \frac{10^3}{10^2} = 10$

$\Delta G^o = -RT \ln K_{eq}$

$= -8.3\times300 \ln 10$

$= -8.3\times300 \times2.3$

$= -5.72\times10^{+3}$ J

= 5.72 KJ

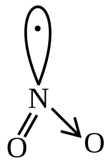
57. The number of molecules or ions from the following, which do not have odd number of electrons are_______

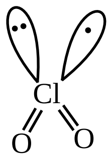
(A) NO_2 (B) ICl_4^- (C) BrF_3 (D) ClO_2

(E) NO_2^+ (F) NO

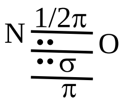
Sol. 3

odd e^-

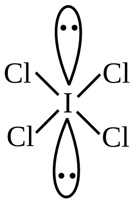


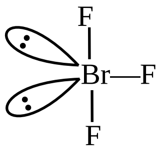


ICl_4^-, BrF_3 and NO_2^+ do not have odd number of electron.



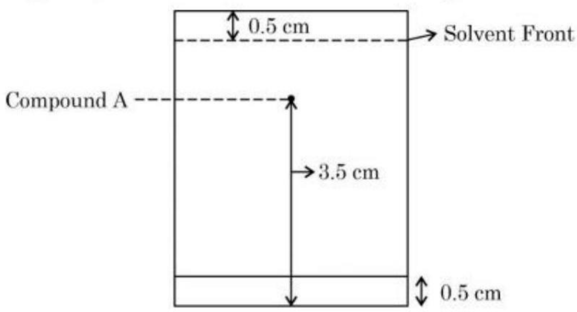
Odd e^- absent





$O=\overset{+}{N}=O$

58. Following chromatogram was developed by adsorption of compound 'A' on a 6 cm TLC glass plate. Retardation factor of the compound 'A' is_______$\times 10^{-1}$

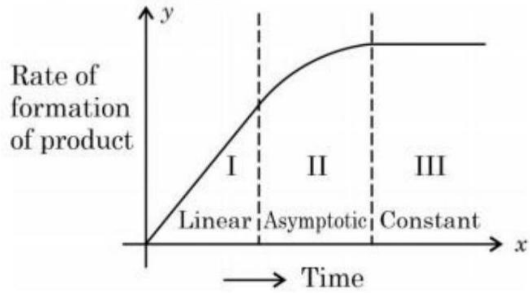


Sol. 6

$$R_f = \frac{\text{Distance moved by the substance from base line}}{\text{Distance moved by the solvent from base line}}$$

$$= \frac{3.0\text{ cm}}{5.0\text{ cm}} = 0.6 \text{ or } 6\times10^{-1}$$

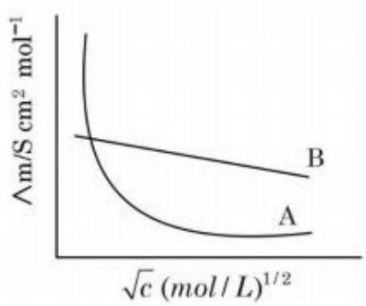
59. For certain chemical reaction $X \rightarrow Y$, the rate of formation of product is plotted against the time as shown in the figure. The number of correct statement/s from the following is______



(A) Over all order of this reaction is one
(B) Order of this reaction can't be determined
(C) In region I and III, the reaction is of first and zero order respectively
(D) In region-II, the reaction is of first order
(E) In region-II, the order of reaction is in the range of 0.1 to 0.9.

Sol. 2
Only option (B) is correctr as order cannot be determined.

60. Following figure shows dependence of molar conductance of two electrolytes on concentration. $\overset{\circ}{\Lambda m}$ is the limiting molar conductivity.



The number of incorrect statement(s) from the following is_________

(A) $\overset{\circ}{\Lambda m}$ for electrolyte A is obtained by extrapolation

(B) For electrolyte B, Λm vs $\sqrt{c}$ graph is a straight line with intercept equal to $\overset{\circ}{\Lambda m}$
(C) At infinite dilution, the value of degree of dissociation approaches zero for electrolyte B.

(D) $\overset{\circ}{\Lambda m}$ for any electrolyte A or B can be calculated using $\lambda°$ for individual ions

Sol. 2
Statement (A) and Statement (C) are incorrect.

Mathematics

Section A

61. Let α and β be real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

(1) $\beta = -8$ (2) $\beta = 8$ (3) $\alpha = 4$ (4) $\alpha = 1$

Sol. **1**

$A^2 = 3A + \alpha I$ …… (1)

and $A^4 = 21A + \beta I$ ……..(2)

Now $A^4 = A^2 . A^2$

$A^4 = (3A + \alpha I) . (3A + \alpha I)$ {from (1)}

$A^4 = 9A^2 + 6\alpha A + \alpha^2 I$ ……..(3)

From (2) and (3)

$9A^2 + 6\alpha A + \alpha^2 I = 21 A + \beta I$

putting value of A^2 from (1)

$9(3A + \alpha I) + 6\alpha A + \alpha^2 I = 21 A + \beta I$

$(27 + 6\alpha)A + (9\alpha + \alpha^2) I = 21A + \beta I$

by comparison

$27 + 6\alpha = 21$ and $9\alpha + \alpha^2 = \beta$

$\Rightarrow 6\alpha = -6$ putting $\alpha = -1$

$\Rightarrow \alpha = -1$ $\therefore \beta = -8$

62. Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and

$$f(x) = \begin{cases} \dfrac{1 - \cos(x^2 - 4px + q^2 + 8q + 16)}{(x - 2p)^4} & , x \neq 2p \\ 0, & , x = 2p \end{cases}$$

$\lim_{x \to 2p^+} [f(x)]$

where $[\cdot]$ denotes greatest integer function, is

(1) 0 (2) -1 (3) 2 (4) 1

Sol. **1**

$$f(x) = \begin{cases} \dfrac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4} & , x \neq 2p \\ 0 & , \quad x = 2p \end{cases}$$

$\because$ x = 2 is a root of equation $x^2 + px + q = 0$

$\therefore 4 + 2p + q = 0$

$\Rightarrow 2p = -q - 4$

$\Rightarrow 4p^2 = (q + 4)^2 = q^2 + 8q + 16$ …….(1)

Now $\lim\limits_{x\to 2p^+} f(x) = \lim\limits_{x\to 2p^+} \dfrac{1-\cos(x^2-4px+4p^2)}{(x-2p)^4}$ (from (1))

$$= \lim_{x\to 2p^+}\left[\frac{1-\cos(x-2p)^2}{\{(x-2p)^2\}^2}\right]$$

$$= \frac{1}{2} \quad \left\{\because \lim_{x\to 0}\frac{1-\cos\theta}{\theta^2} = \frac{1}{2}\right\}$$

$$\therefore \lim_{x\to 2p^+}[f(x)] = \left[\frac{1}{2}\right] = 0$$

63. Let B and C be the two points on the line $y+x=0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y-2x=2$ such that $\triangle ABC$ is an equilateral triangle. Then, the area of the $\triangle ABC$ is

(1) $\frac{10}{\sqrt{3}}$ (2) $3\sqrt{3}$ (3) $2\sqrt{3}$ (4) $\frac{8}{\sqrt{3}}$

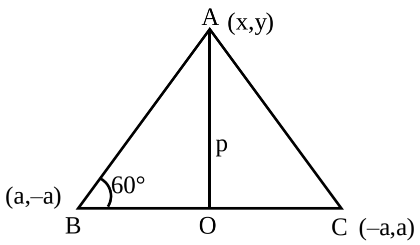
Sol. **4**

Since, A lies on perpendicular bisector of BC, whose equation is

y = x ……..(1)

Now, A is the point of intersection of y = x and y – 2x = 2

$\therefore$ point A, after solving is A(–2, –2)



In ΔAOC $\tan 60° = \frac{p}{OC} \Rightarrow OC = \frac{p}{\sqrt{3}}$ $\{\because OA = p\}$

$\therefore BC = 2 \times OC = \frac{2p}{\sqrt{3}}$

Now, Area of $\Delta ABC = \frac{1}{2} \times BC \times OA$

$= \frac{1}{2} \times \frac{2p}{\sqrt{3}} \times p = \frac{p^2}{\sqrt{3}}$ sq. unit

and $p = OA = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

So, Area of $\Delta ABC = \frac{(2\sqrt{2})^2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$ sq. unit

64. Consider the following system of equations

$\alpha x + 2y + z = 1$

$2\alpha x + 3y + z = 1$

$3x + \alpha y + 2z = \beta$

for some $\alpha, \beta \in \mathbb{R}$. Then which of the following is NOT correct.

(1) It has a solution if $\alpha = -1$ and $\beta \neq 2$

(2) It has a solution for all $\alpha \neq -1$ and $\beta = 2$

(3) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

(4) It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$

Sol. **4**

$$\because\ D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix}$$

$D = \alpha(6-\alpha) + 2(3-4\alpha) + 1(2\alpha^2 - 9)$

$= 6\alpha - \alpha^2 + 6 - 8\alpha + 2\alpha^2 - 9$

$D = \alpha^2 - 2\alpha - 3$

for no solution , D = 0

$\Rightarrow \quad \alpha^2 - 2\alpha - 3 = 0$

$\quad (\alpha + 1)(\alpha - 3) = 0$

$\Rightarrow \quad \alpha = -1, \alpha = 3$

Now,

$$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & \alpha & 2 \end{vmatrix}, \; D_2 = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix} \text{ and } D_3 = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix}$$

if $\alpha = -1$ then

$$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & -1 & 2 \end{vmatrix}, \; D_2 = \begin{vmatrix} -1 & 1 & 1 \\ -2 & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix}, \; D_3 = \begin{vmatrix} -1 & 2 & 1 \\ -2 & 3 & 1 \\ 3 & -1 & \beta \end{vmatrix}$$

$\Rightarrow$ only for $\beta = 2$, $D_1 = 0$, $D_2 = 0$, $D_3 = 0$

$\therefore$ It has no solution if $\alpha = -1$ and $\beta \neq 2$

if $\alpha = 3$

$$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & 3 & 2 \end{vmatrix}, \; D_2 = \begin{vmatrix} 3 & 1 & 1 \\ 6 & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix}, \; D_3 = \begin{vmatrix} 3 & 2 & 1 \\ 6 & 3 & 1 \\ 3 & 3 & \beta \end{vmatrix}$$

$\Rightarrow$ Only for $\beta = 2$, $D_1 = D_2 = D_3 = 0$

$\Rightarrow$ It has no solution for $\beta \neq 2$

$\therefore$ It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

65. Let $y = f(x)$ be the solution of the differential equation $y(x+1)dx - x^2 dy = 0, y(1) = e$. Then $\lim_{x \to 0^+} f(x)$ is equal to

(1) $\frac{1}{e^2}$ (2) e^2 (3) 0 (4) $\frac{1}{e}$

Sol. **3**

$y(x + 1)dx - x^2 dy = 0, \qquad y(1) = e$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y(x+1)}{x^2}$

$\Rightarrow \displaystyle\int \frac{dy}{y} = \int \frac{(x+1)dx}{x^2}$

$\ell ny = \ell nx - \dfrac{1}{x} + c$

$\because y(1) = e$

$\therefore 1 = 0 - 1 + C \Rightarrow C = 2$

Now, $\ell ny = \ell nx - \dfrac{1}{x} + 2$

$\Rightarrow \ell n\left(\frac{y}{x}\right) = 2 - \frac{1}{x}$

$\Rightarrow \frac{y}{x} = e^{2-\frac{1}{x}}$

$\Rightarrow y = x,\ e^{2-\frac{1}{x}}$

So, $\lim\limits_{x\to 0^+} y = \lim\limits_{x\to 0^+} x e^{2-\frac{1}{x}} = 0$

66. The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x}-(2x+3)}, x \in \mathbb{R}$ is

(1) $\mathbb{R} - \{3\}$ (2) $(-1, \infty) - \{3\}$ (3) $(2, \infty) - \{3\}$ (4) $\mathbb{R} - \{-1, 3\}$

Sol. **3**

$f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\ln x} - (2x+3)}$

case (i) $x - 2 > 0 \Rightarrow x > 2$

$x \in (2, \infty)$

case (ii) $x + 1 > 0$ and $x + 1 \neq 1$

$x > -1$, $x \neq 0$

$\therefore\ x_t(-1, 0) \cup (0, \infty)$

case (iii) $x > 0 \Rightarrow x_t (0, \infty)$

case (iv) $e^{2\ell n x} - (2x + 3) \neq 0$

$\Rightarrow$ $x^2 - 2x + 3 \neq 0$

$(x - 3)(x + 1) \neq 0$

$\Rightarrow$ $x \neq 3, x \neq -1$

$\therefore$ from (i) n (ii) n (iii)n (iv)

$x_t (2, \infty) - \{3\}$

67. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is

(1) $\frac{5}{24}$ (2) $\frac{1}{6}$ (3) $\frac{5}{36}$ (4) $\frac{2}{15}$

Sol. **2**

Required probability = $1 - \frac{D_{(15)} + {}^{15}C_1 D_{(14)} + {}^{15}C_2 D_{(3)}}{15!}$

Taking $D_{(15)}$ as $\frac{15!}{e}$

$D_{(14)}$ as $\frac{14!}{e}$

$D_{(13)}$ as $\frac{13!}{e}$

We get $1 - \left(\dfrac{\dfrac{15!}{e} + 15\dfrac{14!}{e} + \dfrac{15\times 14}{2\times 1}\times\dfrac{13!}{e}}{15!}\right)$

$= 1 - \left(\dfrac{1}{e} + \dfrac{1}{e} + \dfrac{1}{2e}\right) = 1 - \dfrac{5}{2e} \simeq 0.08$

68. Let $[x]$ denote the greatest integer $\leq x$. Consider the function $f(x) = \max\{x^2, 1 + [x]\}$. Then the value of the integral $\int_0^2 f(x)dx$ is

(1) $\frac{5+4\sqrt{2}}{3}$ (2) $\frac{4+5\sqrt{2}}{3}$ (3) $\frac{1+5\sqrt{2}}{3}$ (4) $\frac{8+4\sqrt{2}}{3}$

Sol. **1**

f(x) = Max. $\{x^2, 1+[x]\}$

Now, $f(x) = \begin{cases} 1+[x] & 0 \leq x \leq \sqrt{2} \\ x^2 & \sqrt{2} < x \leq 2 \end{cases}$

$\int_0^2 f(x)dx = \int_0^{\sqrt{2}} (1+[x])dx + \int_{\sqrt{2}}^2 x^2 dx$

$= \int_0^1 1dx + \int_1^{\sqrt{2}} 2dx + \int_{\sqrt{2}}^2 x^2 dx$

$= (x)_0^1 + 2(x)_1^{\sqrt{2}} + \dfrac{1}{3}(x^3)_{\sqrt{2}}^2$

$= 1 + 2(\sqrt{2}-1) + \dfrac{1}{3}(8 - 2\sqrt{2})$

$= \dfrac{4\sqrt{2}+5}{3}$

69. For two non-zero complex numbers z_1 and z_2, if $\text{Re}(z_1 z_2) = 0$ and $\text{Re}(z_1 + z_2) = 0$, then which of the following are possible?

A. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$
B. $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) > 0$
C. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) < 0$
D. $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) < 0$

Choose the correct answer from the options given below:

(1) B and D (2) A and B (3) B and C (4) A and C

Sol. **3**

$\text{Re}(z_1z_2) = 0$ and $\text{Re}(z_1 + z_2) = 0$

Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$z_1z_2 = (a_1a_2 - b_1b_2) + i(a_1b_2 + b_1a_2)$

$\because\ \text{Re}(z_1z_2) = a_1a_2 - b_1b_2 = 0$

$\therefore\ a_1a_2 = b_1b_2$ ....(1)

and $\text{Re}(z_1 + z_2) = 0 \Rightarrow a_1 + a_2 = 0$

$\Rightarrow a_2 = -a_1$ ....(2)

from (1) and (2)

$b_1 b_2 = -a_1^2 < 0$

Product of $b_1 b_2$ is Negative.

$\therefore$ $\text{Im}(z_1)$ and $\text{Im}(z_2)$ are also of opposite sign.

70. If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to

(1) 0 (2) 24 (3) 6 (4) 18

Sol. **2**

Vector $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$

are coplanar then

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0 \qquad \Rightarrow \begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow 10\lambda - 2\mu - 56 = 0$

$\Rightarrow 5\lambda - \mu = 28$ ....(1)

also projection of $\vec{a}$ on the $\vec{b}$ is $\sqrt{54}$ units. then

$\vec{a}.\vec{b} = \sqrt{54}$

$$\Rightarrow \frac{-2\lambda + 4\mu - 8}{\sqrt{24}} = \sqrt{54}$$

$\Rightarrow -2\lambda + 4\mu - 8 = 36$

$\Rightarrow -2\lambda + 4\mu = 44$ ....(2)

from (1) and (2)

$$\lambda = \frac{26}{3} \text{ and } \mu = \frac{46}{3}$$

$$\Rightarrow \lambda + \mu = \frac{26+46}{3} = \frac{72}{3} = 24$$

71. Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ and $S = \left\{\theta \in [0,\pi]: f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$. If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to

(1) $\frac{5}{4}$ (2) $\frac{3}{2}$ (3) $\frac{9}{8}$ (4) $\frac{11}{8}$

Sol. **2**

$$f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$$

$$= 3\left(\cos^4\theta + \sin^4\theta\right) - 2\cos^2 2\theta$$

$$= 3\left(1 - \frac{\sin^2 2\theta}{2}\right) - 2\cos^2 2\theta$$

$$=3\left(\frac{2-\sin^2 2\theta}{2}\right)-2\cos^2 2\theta$$

$$=3\left(\frac{1+\cos^2 2\theta}{2}\right)-2\cos^2 2\theta$$

f(θ) = $\frac{3-\cos^2 2\theta}{2}$

$f^1(\theta)$ = $\frac{2}{2}\cos 2\theta \sin 2\theta \times 2$

$f^1(\theta)$ = sin4θ = $\frac{-\sqrt{3}}{2}$

$\theta \in [0, \pi]$

$4\theta \in [0, 4\pi]$

sin 4θ = $\frac{-\sqrt{3}}{2}$

4θ = $\frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

θ = $\frac{\pi}{3}, \frac{5\pi}{12}, \frac{5\pi}{6}, \frac{11\pi}{12}$

4β = $\sum_{\theta \in s} \theta = \frac{\pi}{3} + \frac{5\pi}{12} + \frac{5\pi}{6} + \frac{11\pi}{12} = \frac{4\pi + 5\pi + 10\pi + 11\pi}{12} = \frac{30\pi}{12} = \frac{5\pi}{2}$

β = $\frac{5\pi}{8}$

f(β) = $f\left(\frac{5\pi}{8}\right) = \frac{3-\cos^2\left(\frac{5\pi}{4}\right)}{2} = \frac{3-\frac{1}{2}}{2} = \frac{5}{4}$

72. If p, q and r three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false?

(1) p = T, q = T, r = F
(2) p = T, q = F, r = T
(3) p = F, q = T, r = F
(4) p = T, q = F, r = F

Sol. **3**

$(p \vee q) \vee (\sim p) \vee r) \rightarrow ((\sim q) \vee r)$

T $\rightarrow$ F ≡ F

$\therefore (p \vee q) \wedge ((\sim p) \vee r) \equiv T$ ....(1)

$(\sim q) \vee r \equiv F$ ....(2)

$\Rightarrow \sim q = F, r = F$

$\Rightarrow q = T$

From (1) $p \vee q \equiv T$

$\sim p \vee r \equiv T$

∴ r = F

⇒ ~ p = T

⇒ p = F

$\therefore$ p = F, q = T, r = F

73. Let Δ be the area of the region $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 21, y^2 \le 4x, x \ge 1\}$.
Then $\frac{1}{2}\left(\Delta - 21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to

(1) $2\sqrt{3} - \frac{2}{3}$ (2) $\sqrt{3} - \frac{4}{3}$ (3) $\sqrt{3} - \frac{2}{3}$ (4) $2\sqrt{3} - \frac{1}{3}$

Sol. **2**

Area of Required Region

$$\Delta = 2\left[\int_1^3 2\sqrt{x}\,dx + \int_3^{\sqrt{21}} \sqrt{21 - x^2}\,dx\right]$$

$$= 2\left[2\frac{\left(x^{3/2}\right)_1^3}{(3/2)} + \left\{\frac{(21)}{2}\sin^{-1}\left(\frac{x}{\sqrt{21}}\right) + \frac{x}{2}\sqrt{21 - x^2}\right\}_3^{\sqrt{21}}\right]$$

$$= 2\left[4\sqrt{3} - \frac{4}{3}\right] + (21\sin^{-1}1 + 0) - \left(21\sin^{-1}\left(\frac{3}{\sqrt{21}}\right) + 3\sqrt{12}\right)$$

$$\Delta = 8\sqrt{3} - \frac{8}{3} + \frac{21\pi}{2} - 6\sqrt{3} - 21\sin^{-1}\sqrt{\frac{3}{7}}$$

$$\Delta = 2\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$$

Now,

$$\frac{1}{2}\left(\Delta_1 - 21\sin^{-1}\frac{2}{\sqrt{7}}\right) = \frac{1}{2}\left[2\sqrt{3} + \frac{21}{2}\pi - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right) - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right]$$

$$= \frac{1}{2}\left[2\sqrt{3} + \frac{21}{2}\pi - \frac{8}{3} - 21\sin^{-1}1\right]$$

$$\left\{\text{using}\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1 - y^2} + y\sqrt{1 - x^2}\right\}\right\}$$

$$= \frac{1}{2}\left[2\sqrt{3} - \frac{8}{3}\right] = \sqrt{3} - \frac{4}{3}$$

74. A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line $x + y = 1$, if this ray intersects x-axis at Q, then the abscissa of Q is

(1) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$ (2) $\frac{2}{3+\sqrt{3}}$ (3) $\frac{2}{(\sqrt{3}-1)}$ (4) $\frac{2}{3-\sqrt{3}}$

Sol. **2**

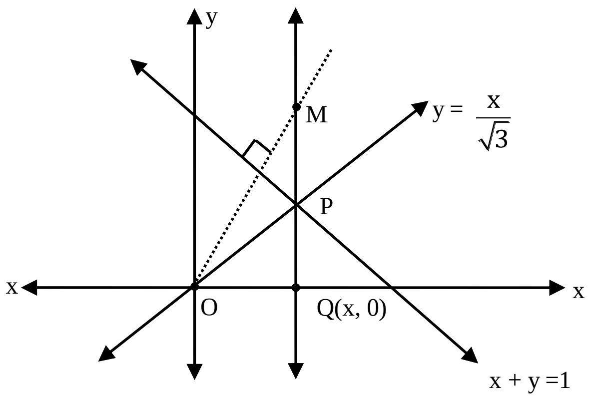
Equation of ray is

$$y = \frac{1}{\sqrt{3}}x \qquad \text{……..(1)}$$

Image of 0(0, 0) in the line x + y = 1 is lies on reflected ray.

$$\frac{x - 0}{1} = \frac{y - 0}{1} = -2\frac{(0 + 0 - 1)}{2}$$

$\Rightarrow$ M (1, 1)



$\therefore$ Point of Intersection of lines $y = \frac{x}{\sqrt{3}}$ and $x + y = 1$ is p (x, y)

$\therefore p\left(\frac{3-\sqrt{3}}{2}, \frac{\sqrt{3}-1}{2}\right)$

Now Reflected Ray is same as line passing through PM.

$\therefore$ Slope of PM = $\dfrac{\frac{\sqrt{3}-1}{2}-1}{\frac{3-\sqrt{3}}{2}-1} = \dfrac{\sqrt{3}-3}{1-\sqrt{3}} = \sqrt{3}$

Equation of PM whose slope is $\sqrt{3}$ and passing through M (1, 1).

$y - 1 = \sqrt{3}(x - 1)$

$y = \sqrt{3}\ x + \left(-\sqrt{3}+1\right)$

$\because$ ray, Intersects x-axis at $\alpha(x, 0)$

$\therefore$ y = 0

$\Rightarrow \sqrt{3}x = -1\left(-\sqrt{3}+1\right) \Rightarrow \sqrt{3}x = \sqrt{3}-1$

$\Rightarrow x = 1 - \frac{1}{\sqrt{3}}$

$x = \frac{\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}+1}{\left(\sqrt{3}+1\right)} = \frac{2}{3+\sqrt{3}}$

$\therefore$ abscissa of α is $\frac{2}{3+\sqrt{3}}$

75. Let $A = \left\{(x, y) \in \mathbb{R}^2 : y \geq 0, 2x \leq y \leq \sqrt{4-(x-1)^2}\right\}$ and

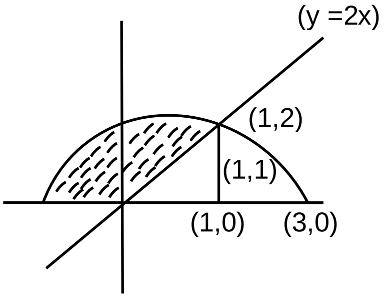
$$B = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} : 0 \leq y \leq \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}.$$

Then the ratio of the area of A to the area of B is

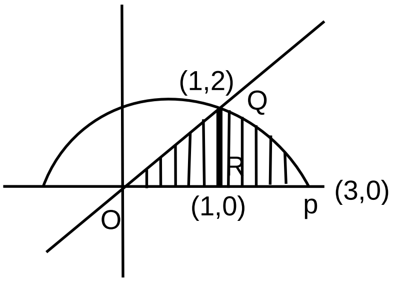
(1) $\frac{\pi+1}{\pi-1}$ (2) $\frac{\pi}{\pi-1}$ (3) $\frac{\pi-1}{\pi+1}$ (4) $\frac{\pi}{\pi+1}$

Sol. **3**

A =



B =



$y^2 = 4 - (x - 1)^2$

$(x - 1)^2 + y^2 = 2^2$

$y = 2x$

$(x - 1)^2 + 4x^2 = 4$

$x^2 + 1 - 2x + 4x^2 = 4$

$5x^2 - 2x - 3 = 0$

$5x^2 - 5x + 3x - 3 = 0$

$5x(x - 1) + 3(x - 1) = 0$

$x = 1, -3/5$

For B : req. area = ar (ΔDRQ) + ar (RPQ)

$$= \frac{1}{2} \times 1 \times 2 + \int_1^3 \sqrt{4 - (x-1)^2}\,dx$$

$$= 1 + \left[\left(\frac{x-1}{2}\right)\sqrt{4-(x-1)^2} + \frac{4}{2}\sin^{-1}\left(\frac{x-1}{2}\right)\right]_1^3$$

$= 1 + 2\sin^{-1} 1 = 1 + \pi$ ....(1)

For A : req. area = area of semi circle – shaded area of B

$$= \frac{\pi r^2}{2} - (1 + \pi)$$

$$= \frac{\pi \times 4}{2} - (1 + \pi) \qquad \{\because r = 2\}$$

$A = \pi - 1$ .... (2)

$$\therefore \frac{A}{B} = \frac{\pi - 1}{\pi + 1}$$

76. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation

(1) $49x^2 - 245x + 250 = 0$ (2) $7x^2 + 245x - 250 = 0$

(3) $7x^2 - 245x + 250 = 0$ (4) $49x^2 + 245x + 250 = 0$

Sol. **1**

$14x^2 - 31x + 3\lambda = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$

and $35x^2 - 53x + 4\lambda = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \gamma \end{array}$

Now, one root is common then

$\therefore\ 14\alpha^2 - 31\alpha + 3\lambda = 0$ …….. (1)

$35\alpha^2 - 53\alpha + 4\lambda = 0$ ……… (2)

$$\frac{\alpha^2}{-124\lambda + 159\lambda} = \frac{-\alpha}{56\lambda - 105\lambda} = \frac{1}{343}$$

$$\Rightarrow \frac{\alpha^2}{35\lambda} = \frac{\alpha}{49\lambda} = \frac{1}{343}$$

$\Rightarrow \alpha = \frac{\lambda}{7}$ {from (ii) and (iii)}

and $\alpha^2 = \frac{35\lambda}{343}$

$$\Rightarrow \frac{\lambda^2}{49} = \frac{35\lambda}{343}$$

$\lambda^2 - 5\lambda = 0$

$\lambda(\lambda - 5) = 0$

$\lambda = 0, \lambda = 5 \quad \Rightarrow \alpha = 5/7$

not possible $\therefore$ only $\lambda = 5$ possible

Now, $\alpha + \beta = \frac{31}{14}$, $\alpha\beta = \frac{3\lambda}{14}$, $\alpha + \gamma = \frac{53}{35}$, $\alpha\gamma = \frac{4\lambda}{35}$

$\therefore\ \beta = \frac{3}{2}$ and $\gamma = \frac{4}{5}$

Now equation having roots $\left(\frac{3\alpha}{\beta}, \frac{4\alpha}{\gamma}\right) = \left(\frac{10}{7}, \frac{25}{7}\right)$ is

$$x^2 - \frac{35}{7}x + \frac{250}{49} = 0$$

$\Rightarrow 49x^2 - 245x + 250 = 0$

77. Let the tangents at the points $A(4, -11)$ and $B(8, -5)$ on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$, intersect at the point C. Then the radius of the circle, whose centre is C and the line joining A and B is its tangent, is equal to

(1) $2\sqrt{13}$ (2) $\sqrt{13}$ (3) $\frac{3\sqrt{3}}{4}$ (4) $\frac{2\sqrt{13}}{3}$

Sol. **4**

Equation of line AB is

y + 5 = $\left(\frac{-5+11}{8-4}\right)(x-8)$

⇒ y + 5 = $\frac{3}{2}$(x – 8) Þ 2y + 10 = 3x – 24

3x – 2y – 34 = 0 ……..(i)

Let C be (h, k) then equation of AB

hx + ky – $\frac{3}{2}$ (x + h) + 5 (y + k) – 15 = 0

x(h – $\frac{3}{2}$) + y (k + 5) $-\frac{3}{2}$h + 5k – 15 = 0 ……..(ii)

Now, by comparing (i) and (ii)

$$\frac{h-\frac{3}{2}}{3}=\frac{k+5}{-2}=\frac{-\frac{3}{2}h+5k-15}{-34}$$

after solving centre C is

(h, k) = $\left(8,\frac{-28}{3}\right)$

and radius of circle is

$$r=\left|\frac{3(8)-2\left(\frac{-28}{3}\right)-34}{\sqrt{9+4}}\right|=\left|\frac{24+2\frac{56}{3}-34}{\sqrt{13}}\right|$$

$$r=\left|\frac{26}{3\sqrt{13}}\right| = \frac{2\sqrt{13}}{3}$$

78. Let $f(x)=x+\frac{a}{\pi^2-4}\sin x+\frac{b}{\pi^2-4}\cos x, x\in\mathbb{R}$ be a function which satisfies $f(x)=x+\int_0^{\pi/2}\sin(x+y)f(y)dy$. Then (a+b) is equal to

(1) –2π(π–2) (2) –2π(π+2) (3) –π(π–2) (4) –π(π+2)

Sol. **2**

$$f(x)=x+\int_0^{\frac{\pi}{2}}(\sin x\cos y+\cos x\sin y)f(y)dy$$

$$f(x)=x+\int_0^{\frac{\pi}{2}}(\cos y\, f(y)dy)\sin x+(\sin y\, f(y)dy)\cos x) \quad ………(1)$$

given : f(x) = x + $\frac{a}{\pi^2-4}\sin x+\frac{b}{\pi^2-4}\cos x$ …….(2)

by comparing (1) and (2)

$$\Rightarrow \frac{a}{\pi^2-4} = \int_0^{\frac{\pi}{2}} \cos y\, f(y)dy \qquad ....(3)$$

and $$\frac{b}{\pi^2-4} = \int_0^{\frac{\pi}{2}} \sin y\, f(y)dy \qquad ....(4)$$

adding (3) and (4)

$$\frac{a+b}{\pi^2-4} = \int_0^{\frac{\pi}{2}} (\sin y + \cos y) f(y)dy \qquad ....(5)$$

$$\frac{a+b}{\pi^2-4} = \int_0^{\frac{\pi}{2}} (\sin y + \cos y) f\left(\frac{\pi}{2} - y\right)dy \qquad ....(6)$$

Additing (5) and (6)

$$\frac{2(a+b)}{\pi^2-4} = \int_0^{\frac{\pi}{2}} (\sin y + \cos y)\left(\frac{\pi}{2} + \frac{a+b}{\pi^2-4}(\sin y + \cos y)\right)dy$$

$$= \pi + \frac{a+b}{\pi^2-4}\left(\frac{\pi}{2}+1\right)$$

$\Rightarrow$ a + b = – 2π (π + 2)

79. Let $f: \mathbf{R} \to \mathbf{R}$ be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then

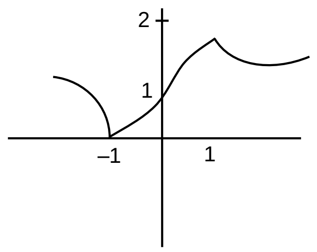
(1) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

(2) $f(x)$ is one-one in $(-\infty, \infty)$

(3) $f(x)$ is many-one in $(-\infty, -1)$

(4) $f(x)$ is many-one in $(1, \infty)$

Sol. **1**



$$f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$$

$$f(x) = 1 + \frac{2}{x + \frac{1}{x}}$$

Clearly, f(x) is one – one in [1, ∞]

but not in (-∞, ∞)

80. Three rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X, respectively, then $10(\mu^2+\sigma^2)$ is equal to

(1) 250 (2) 25 (3) 30 (4) 20

Sol. **4**

Total Apple = 10, Rotten apple = 3, good apple = 7

Prob. of rotten apple (p) = $\frac{3}{10}$

Prob. of good apple (q) = $\frac{7}{10}$

x $\rightarrow$ Number of rotten apples

here x = 0, 1, 2, 3

p(x = 0) = ${}^4C_0\left(\frac{3}{10}\right)^0\times\frac{7}{10}\times\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7}=\frac{1}{6}$

p(x = 1) = ${}^4C_1\left(\frac{3}{10}\right)\times\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}=\frac{1}{2}$

p(x = 2) = ${}^4C_2\left(\frac{3}{10}\times\frac{2}{9}\right)\times\frac{7}{8}\times\frac{6}{7}=\frac{3}{10}$

p(x = 3) = ${}^4C_3\left(\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}\right)\times\frac{7}{7}=\frac{1}{30}$

x_i	0	1	2	3
p_i	$\frac{35}{210}$	$\frac{105}{210}$	$\frac{3}{10}$	$\frac{1}{30}$

Now,

$\mu = \sum p_i x_i = \frac{1}{6}\times 0+\frac{1}{2}\times 1+2\times\frac{3}{10}+3\times\frac{1}{30}=\frac{6}{5}$

and $\sigma^2 = \sum p_i x_i^2-\mu^2=\frac{1}{2}+\frac{3}{10}\times 4+\frac{1}{30}\times 9-\frac{36}{25}=\frac{14}{25}$

$\therefore$ $10(\mu^2+\sigma^2) = 10\left(\frac{36}{25}+\frac{14}{25}\right)$

$= 10\times\left(\frac{50}{25}\right)=10\times 2$

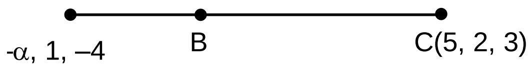
$= 20$

Section B

81. Let the co-ordinates of one vertex of $\triangle ABC$ be $A(0,2,\alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. For $\alpha\in\mathbb{Z}$, if the area of $\triangle ABC$ is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to

Sol. **9**

A $(0, 2, \alpha)$



$$\left|\frac{1}{2}.2\sqrt{21}\begin{vmatrix} i & j & k \\ \alpha & 1 & \alpha+4 \\ 5 & 2 & 3 \end{vmatrix}\frac{1}{\sqrt{25+4+9}}\right| = 21$$

$$\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2} = \sqrt{21}\sqrt{38}$$

$12\alpha^2 + 80\alpha + 450 = 798$

$12\alpha^2 + 80\alpha - 398 = 0$

$\alpha = 3 \Rightarrow \alpha^2 = 9$

82. Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies the relation $f(x+y) = f(x) + f(y) - 1, \forall x, y \in \mathbb{R}$. If $f'(0) = 2$, then $|f(-2)|$ is equal to

Sol. **3**

Given $f(x + y) = f(x) + f(y) - 1 \ \forall \ x, y \in IR$ and $f'(0) = 2$

Partial differentiate w.r.t x

$\Rightarrow f'(x + y)\ f'(x)$

for $x = 0$

$f'(y) = f'(0) = 2$

on Integrating

$\Rightarrow f(y) = 2y + c$ ……...(2)

for $y = 0$

$\Rightarrow f(0) = C$ …….(3)

Put $x = y = 0$ in (1)

$\Rightarrow f(0) = f(0) + f(0) - 1$

$\Rightarrow f(0) = 1$ ……..(4)

from (3) & (4)

$c = 1$

$\Rightarrow f(y) = 2y + 1$

$\Rightarrow f(-2) = -4 + 1 = -3$

$\therefore |f(-2)| = 3$

83. Suppose f is a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{N}$ and $f(1) = \frac{1}{5}$. If $\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to

Sol. **10**

$f(x + y) = f(x) + f(y) \ \forall \ x, y \in N$ and $f(1) = \frac{1}{5}$

for $x = y = 1$

$f(2) = f(1) + f(1) = 2f(1)$

$f(3) = f(2+1) = f(2) + f(1) = 3f(1)$

In General

$f(n) = nf(1) = \frac{n}{5}$

$$\sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$$

$$\Rightarrow \sum_{n=1}^{m}\frac{n}{5n(n+1)(n+2)}=\frac{1}{12}$$

$$\Rightarrow \frac{1}{5}\sum_{n=1}^{m}\frac{1}{(n+1)(n+2)}=\frac{1}{12}$$

$$\Rightarrow \sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)=\frac{5}{12}$$

$$\Rightarrow \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\text{.......}+\left(\frac{1}{m+1}-\frac{1}{m+2}\right)=\frac{5}{12}$$

$$\Rightarrow \frac{1}{2}-\frac{1}{m+2}=\frac{5}{12}$$

$$\Rightarrow \frac{1}{m+2}=\frac{1}{2}-\frac{5}{12}=\frac{1}{12}$$

$\Rightarrow$ m = 10

84. Let the coefficients of three consecutive terms in the binomial expansion of $(1+2x)^n$ be in the ratio $2:5:8$. Then the coefficient of the term, which is in the middle of these three terms, is

Sol. **1120**

Let r + 1, r + 2 and r + 3 be three consecutive terms

$$\frac{{}^{n}C_{r}2^{r}}{{}^{n}C_{r+1}2^{r+1}}=\frac{2}{5}$$

$$\Rightarrow \frac{r+1}{n-r}=\frac{4}{5} \qquad \text{……(1)}$$

Also,

$$\frac{{}^{n}C_{r+1}2^{r+1}}{{}^{4}C_{r+2}2^{r+2}}=\frac{5}{8}$$

$$\Rightarrow \frac{r+2}{n-r-1}=\frac{5}{4} \qquad \text{…….(2)}$$

on solving (1) & (2), we get

n = 8, r = 3

Here n = 8 (even)

middle term = r + 2 = 3 + 2 = 5

coefficient of T_5 = ${}^{8}C_{4}2^{4}=70(16)=1120$

85. Let $a_1, a_2, a_3, \ldots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24 , then $a_1a_9+a_2a_4a_9+a_5+a_7$ is equal to

Sol. **60**

Let first term of G.P be a with common ratio r

Given : $a_4 \,.\, a_6 = 9$

$a_5 + a_7 = 24$

$a_4 = ar^3$, $a_5 = ar^4$, $a_6 = ar^5$, $a_7 = ar^6$

$a_4 \,.\, a_6 = a^2r^8 = 9$

$\Rightarrow \quad ar^4 = 3$

$\quad a_5 = 3$

$\therefore \quad a_7 = 24 - 3 = 21$

$\Rightarrow \quad \dfrac{a_7}{a_5} = r^2 = 7$

$\Rightarrow \quad r = \sqrt{7}$, $a = \dfrac{3}{49}$

$a_1 a_9 + a_2 a_4 a_9 + \quad a_5 + a_7 = a_1 a_9 + (ar)(ar^3) a_9 + 24$

$= a_1 a_9 + a_1(ar^4)a_9 + 24$

$= a_1 a_9 (1 + a_5) + 24 = (ar^4)^2 (4) + 24$

$= 36 + 24 = 60$

86. Let the equation of the plane P containing the line $x + 10 = \frac{8-y}{2} = z$ be $ax + by + 3z = 2(a + b)$ and the distance of the plane P from the point $(1,27,7)$ be c. Then $a^2 + b^2 + c^2$ is equal to

Sol. **355**

Given equation of plane is

$ax + by + 3z = 2(a + b)$ ……..(1)

It containing the line

$\dfrac{x-(-10)}{1} = \dfrac{y-8}{-2} = \dfrac{z-0}{1}$

$\therefore$ plane (1) must passes through (–10, 8, 0) and parallel to 1, –2, 1

Hence,

$\quad a(-10) + 8b = 2a + 2b$

$\Rightarrow \quad 12a - 6b = 0$ ……..(2)

and $\quad a - 2b + 3 = 0$ ……..(3)

on solving (2) and (3), we get

$\quad b = 2, a = 1$

$\therefore$ equation of the plane is

$\quad x + 2y + 3z = 6$ ……..(4)

c is perpendicular distance from (1, 27, 7) to the plane (4)

$\Rightarrow c = \left|\dfrac{1+2\times27+3\times7-6}{\sqrt{1^2+2^2+3^2}}\right| = \left|\dfrac{70}{\sqrt{14}}\right| = \dfrac{10\sqrt{7}}{\sqrt{2}}$

Now, $a^2 + b^2 + c^2 = 1 + 4 + \dfrac{700}{2} = \dfrac{710}{2} = 355$

87. If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to

Sol. **1**

For $\left(\alpha x^3 + \dfrac{1}{\beta x}\right)^{11}$

$T_{r+1} = {}^{11}C_r (\alpha x^3)^{11-r} \left(\dfrac{1}{\beta x}\right)^r$

$= {}^{11}C_r \alpha^{11-r} \beta^{-r} x^{33-4r}$

Coefficient of $x^9 = {}^{11}C_6 \alpha^{11-6} \beta^{-6}$

$= {}^{11}C_6 \alpha^5 \beta^{-6}$

For $\left(\alpha x - \dfrac{1}{\beta x^3}\right)^{11}$

$T_{r+1} = {}^{11}C_r (\alpha x)^{11-r} \left(\dfrac{-1}{\beta x^3}\right)^r$

$= (-1)^r\ {}^{11}C_r \alpha^{11-r} \beta^{-r} x^{11-4r}$

coefficient of $x^{-9} = -\ {}^{11}C_5 \alpha^6 \beta^{-5}$

$\Rightarrow {}^{11}C_6 \alpha^5 \beta^{-6} = {}^{11}C_5 \alpha^6 \beta^{-5}$

$\Rightarrow \alpha\beta = -\dfrac{{}^{11}C_6}{{}^{11}C_5} = -1$

$\therefore (\alpha\beta)^2 = 1$

88. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a} - \vec{b} + \vec{c}$, $\lambda\vec{a} - 3\vec{b} + 4\vec{c}$, $-\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, then λ is equal to

Sol. **2**

$\overrightarrow{AB} = (\lambda\vec{a} - 3\vec{b} + 4\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$

$= (\lambda - 1)\ \vec{a} - 2\vec{b} + 3\vec{c}$

$\overrightarrow{AC} = (-\vec{a} + 2\vec{b} - 3\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$

$= -2\vec{a} + 3\vec{b} - 4\vec{c}$

$\overrightarrow{AD} = (2\vec{a} - 4\vec{b} + 6\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$

$= \vec{a} - 3\vec{b} + 5\vec{c}$

For coplanar vectors

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow 3\lambda - 6 = 0$

$\therefore \lambda = 2$

89. Five digit numbers are formed using the digits 1, 2, 3,5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1 . Then the serial number of 35337 is

Sol. **1436**

Number starting with 7 = $7\underset{5}{\underset{\uparrow}{\_}}\,\underset{5}{\underset{\uparrow}{\_}}\,\underset{5}{\underset{\uparrow}{\_}}\,\underset{5}{\underset{\uparrow}{\_}} = 625$

Number starting with 5 = $5\underset{5}{\underset{\uparrow}{\_}}\,\underset{5}{\underset{\uparrow}{\_}}\,\underset{5}{\underset{\uparrow}{\_}}\,\underset{5}{\underset{\uparrow}{\_}} = 625$

Number starting with 37 = $37\underset{5}{\underset{\uparrow}{\_}}\ \underset{5}{\underset{\uparrow}{\_}}\ \underset{5}{\underset{\uparrow}{\_}}$ = 125

Number starting with 357 = $357\underset{5}{\underset{\uparrow}{\_}}\ \underset{5}{\underset{\uparrow}{\_}}$ = 25

Number starting with 355 = $355\_\ \_$ = 25

Number starting with 3537 = $3537\ \_ = 5$

Number starting with 3535 = $3535\ \_ = 5$

Number starting with 35337 = 1

Total = 1436

Therefore, the serial number of 35337 is 1436

90. If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72^{th} number is

Sol. **32**

Number of six digit number starting with 1 is 1 ………….. = ${}^8C_5 = 56$

As remaining five digits can be selected from 8 digits that are greater than (i.e., 2, 3, 4, 5, 6, 7, 8)

Number of six digit number starting with 23 ………. = ${}^6C_4 = 15$

Total = 56 + 15 = 71

Now, 72^{nd} number = 245678

$\therefore$ sum of the digits = 2 + 4 + 5 + 6 + 7 + 8 =32